Spin Effects on the Plasma Oscillations of an Electron Gas in a Magnetic Field*

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A dispersion relation—which includes the effects of spin—for the longitudinal oscillations of an electronic plasma in a magnetic field is calculated. In the long-wavelength and low-temperature limit, explicit formulas for the frequencies are obtained, and it is suggested that the result may be useful for the experimental determination of the product of the electronic g factor and the effective mass.

INTRODUCTION

THE extension of the classical analysis of the longitudinal oscillations of an electron gas to the case for which quantum effects are important has been reported on in many studies.[1] Not unexpectedly, the introduction of an external magnetic field complicates the analysis enormously and only recently has a completely quantum-mechanical treatment of this phenomenon been given by Zyryanov.[2] Under the conditions for which most quantum plasmas are available in the laboratory, the effects of the electronic spin are negligible, and in particular they were not treated in the above reference.

The purpose of this paper is to show that under certain experimentally obtainable conditions, the effects of electron spin can modify the results of Zyryanov appreciably and in turn lead to formulas which, for example, may be useful for measuring the product of the anomalous electronic g factor and the effective mass. In particular, it is shown that for a degenerate electron gas of fairly low density in the presence of a strong magnetic field, the spin dependence becomes important and is very sensitive to variations in both the field strength and the parameter gm^*/m. In order that this spin structure be observable, certain inequalities involving the density, the temperature, the energy gap, and the magnetic-field strength must be satisfied. In the concluding section it is argued that for n-type InSb and InAs, these conditions can all be satisfied for currently available field strengths, and that therefore the present results can be used to measure the parameter gm^*/m by reflection experiments.[3,4]

ANALYSIS

Since the present calculation is directed towards low-density electron gases, it is reasonable to assume that exchange effects are negligible. The linearized equation for the perturbation in the one-particle density matrix thus becomes

$$i\hbar\frac{\partial}{\partial t}\langle k'|\rho(t)|k\rangle=[\epsilon_{k'}-\epsilon_k]\langle k'|\rho(t)|k\rangle +[h(\epsilon_k)-h(\epsilon_{k'})]\langle k'|H'(\rho)|k\rangle, \quad (1)$$

where the matrix elements of $H'(\rho)$ are given by

$$\langle k'|H'(\rho)|k\rangle=\int dqdq'\langle k'q|V|kq'\rangle\langle q'|\rho|q\rangle, \quad (2)$$

and where the symbols k, q represent all quantum numbers required to specify the single-particle states, and V is the Coulomb repulsion between two electrons. The quantity $h(\epsilon_k)$ is the Fermi-Dirac distribution and ϵ_k is the Hartree energy associated with the state $|k\rangle$. In particular, assuming that the magnetic field $\mathbf{H}$ is along the z axis, these states may be taken to be[5]

$$|n,k_x,k_zs\rangle=\alpha^{-1/2}\exp\{i(xk_x+zk_z)\} \times\theta_n[(y/\alpha)+\alpha k_x]\chi(s), \quad (3)$$

where θ_n is a normalized harmonic-oscillator function and α is defined by $\alpha^2=\hbar/m\omega_c$, where ω_c is the cyclotron frequency and given by $\omega_c=eH/mc$. The energy associated with this state is

$$\epsilon_{n,k_z,s}=(\hbar^2k_z^2/2m)+\hbar\omega_c(n+\tfrac{1}{2})-\hbar\omega_c gs, \quad (4)$$

where n ranges over all nonnegative integers, and the spin variable s takes on the values $\pm\frac{1}{2}$. In these and all of the remaining formulas, it is to be understood that except in the spin energy, m is always to be replaced by the effective mass m^*.

Following Zyryanov[2] we shall approximate the matrix element for the Coulomb potential by the formula

$$\begin{aligned}\langle n',k_x',k_z',s;m,q_x,q_zs'|V|n,k_x,k_z,s;m',q_x',q_z's'\rangle \\ \cong\{2e^2(2\pi)^3/[(k_x-k_x')^2+(k_z-k_z')^2]\} \\ \times\delta(k_x'+q_x-k_x-q_x')\delta(k_z'+q_z-k_z-q_z') \\ \times F_{n'n}(\alpha[k_x'-k_x])F_{m'm}(\alpha[q_x'-q_x]), \quad (5)\end{aligned}$$

where $F_{n'n}$ is the overlap integral of two harmonic-oscillator functions and may be expressed in terms of the associated Laguerre polynomial,[6]

$$L_n^m(x)=(1/n!)e^xx^{-m}(d^n/dx^n)(e^{-x}x^{n+m}),$$

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[1] See for example, D. Bohm and D. Pines, Phys. Rev. **92**, 609 (1953), and other references cited there.

[2] P. S. Zyryanov, Zh. Eksperim. i Teor. Fiz. **40**, 1065, 1353 (1961) [translations: Soviet Phys.—JETP **13**, 751, 953 (1961)].

[3] W. G. Spitzer and H. Y. Fan, Phys. Rev. **106**, 882 (1957).

[4] As noted in Ref. 2, in the long-wavelength limit, the roots of the dispersion relation coincide with the zeros of the transverse dielectric constant.

[5] L. D. Landau, Z. Physik **64**, 629 (1930).

[6] A. Erdelyi, W. Magnus, F. Oberhettinger, and F. G. Tricomi, *Higher Transcendental Functions* (McGraw-Hill Book Company, Inc., New York, 1953), Vol. 2, p. 188.

by the formula

$$F_{n'n}(x)=\left[\frac{n'!}{n!}\right]^{1/2}\exp\left\{-\frac{x^2}{4}\right\}\times\left(-\frac{x}{2^{1/2}}\right)^{n-n'}L_{n'}^{n-n'}\left(\frac{x^2}{2}\right);\quad(n'\leqslant n),\quad(6)$$

with an obvious modification for the case $n\leqslant n'$. On substituting Eqs. (2) and (5) into Eq. (1) and carrying out a Fourier transform in time, one finds in a straightforward way that the allowable frequencies of oscillations of the electron gas are given by those values of ω which satisfy the dispersion relation

$$1=\frac{e^2}{2\pi^2\alpha^2}\sum_{n,n',s}\int dk_z\frac{F_{n'n}^2(\alpha p_x)}{p_x^2+p_z^2}\times\frac{h(\epsilon_{n,k_z,s})-h(\epsilon_{n',k_z+p_z s})}{\hbar\omega+\epsilon_{n,k_z s}-\epsilon_{n'k_z+p_z s}}.\quad(7)$$

For the special case that the single-particle energies are independent of the spin, this reduces to the corresponding formula of Zyryanov.

In the zero-temperature limit, the Fermi function h has the form

$$h(\epsilon)=1,\quad\epsilon\leqslant\mu,$$
$$=0,\quad\epsilon>\mu,\quad(8)$$

where μ is the Fermi energy, and for this case the dispersion relation can be considerably simplified. The limits on the k_z integration for the two terms of Eq. (7) become then, respectively,

$$-(2^{1/2}/\alpha)[n_0(s)-n]^{1/2}<k_z<(2^{1/2}/\alpha)[n_0(s)-n]^{1/2},$$
$$-p_z-(2^{1/2}/\alpha)[n_0(s)-n']^{1/2}<k_z<-p_z+(2^{1/2}/\alpha)[n_0(s)-n']^{1/2},\quad(9)$$

where $n_0(s)$ is defined in terms of the Fermi energy by

$$n_0(s)=(\mu/\hbar\omega_c)-\tfrac{1}{2}+g(m^*/m)s,\quad(10)$$

and is related to the electron density N_0 by the formula

$$N_0=\frac{1}{2^{1/2}\pi^2\alpha^3}\sum_s\sum_{n=0}^{[n_0(s)]}[n_0(s)-n]^{1/2}.\quad(11)$$

The symbol $[n_0(s)]$ represents the largest positive integer not exceeding $n_0(s)$. Carrying out this integration one obtains

$$1=\frac{e^2}{\pi\hbar\omega_c\alpha}\sum_s\sum_{n,n'}^{[n_0(s)]}\frac{F_{n'n}^2(p_x)}{p_z(p_x^2+p_z^2)}\times\left\{\ln\frac{\omega+n-n'+2^{1/2}p_z[n_0(s)-n]^{1/2}-\frac{1}{2}p_z^2}{\omega+n-n'-2^{1/2}p_z[n_0(s)-n]^{1/2}-\frac{1}{2}p_z^2}+\ln\frac{\omega+n-n'-2^{1/2}p_z[n_0(s)-n]^{1/2}+\frac{1}{2}p_z^2}{\omega+n-n'+2^{1/2}p_z[n_0(s)-n]^{1/2}+\frac{1}{2}p_z^2}\right\},\quad(12)$$

where ω and p_z are in units of ω_c and $1/\alpha$, respectively, and where ω may now be taken to be real since there is no damping at zero temperature.

By analogy to the classical case, one expects that except for extraordinarily large magnetic fields, the roots of Eq. (12) will be of the order of the plasma frequency with a very small correction term proportional to $(p_x^2+p_z^2)$. Making use of the fact that the present collective description is valid only for the case that the wavelengths of the disturbance are large compared to the mean interparticle spacing, it follows that for densities of order 10^{17}/cc, such p^2-dependent corrections are negligible and thus one need only evaluate the dispersion relation in the long-wavelength limit. Therefore, keeping terms only to order p_x^2 and p_z^2, Eq. (12) may be cast into the simpler form

$$1=\frac{e^2}{\pi}\frac{2^{3/2}}{\hbar\omega_c\alpha}\sum_s\sum_{n,n'}^{[n_0(s)]}\left\{\frac{F_{n'n}^2(p_x)}{p_x^2+p_z^2}\times\left[\frac{[n_0(s)-n]^{1/2}-[n_0(s)-n']^{1/2}}{\omega+n-n'}+\frac{1}{2}p_z^2\frac{[n_0(s)-n]^{1/2}+[n_0(s)-n']^{1/2}}{(\omega+n-n')^2}\right]\right\},\quad(13)$$

where to this order $F_{n'n}^2(p_x)$ is given by

$$F_{n'n}^2(p_x)\cong(\tfrac{1}{2}p_x^2)^{n-n'}\frac{n!}{n'![(n-n')!]^2};\quad n'\leqslant n\quad(14)$$

with a corresponding formula for $n'\geqslant n$. An examination of Eqs. (13)–(14) shows that in this long-wavelength limit there are no electronic transitions between Landau levels for disturbances which propagate parallel to the direction of the magnetic field. For propagation perpendicular to the field, on the other hand, the electrons must make transitions between adjacent Landau levels, and this means there will be no such waves until the parameter $n_0(1/2)$ is larger than unity. More specifically, combining Eqs. (13)–(14) and making use of Eq. (11), one finds that in this long-wavelength limit the dispersion relation takes on the form

$$1=\cos^2\theta\frac{\omega_p^2}{\omega^2}+\frac{\sin^2\theta\omega_p^2}{\omega^2-\omega_c^2}[1-\sum_s\Delta(s)],\quad(15)$$

where θ is the angle between $\mathbf{p}$ and $\mathbf{H}$, ω_p is the plasma frequency

$$\omega_p^2=4\pi N_0e^2/m^*,$$

and the spin-dependent function $\Delta(s)$ is given by

$$\Delta(s)=\frac{[n_0(s)]+1}{2^{1/2}\pi^2\alpha^3N_0}[n_0(s)-[n_0(s)]]^{1/2};\quad n_0(s)\geqslant1,$$
$$=1;\quad n_0(s)<1.\quad(16)$$

An examination of Eq. (15) shows that for propagation along the magnetic field, the only solution of the dispersion relation is the classical plasma frequency ω_p.

This is physically reasonable, since for these longitudinal waves, the perturbed particle motions are along the direction of the field and thus are not modified by it. On the other hand, for propagation at right angles to the field, the dispersion relation shows a fairly sensitive magnetic field variation because of the spin-dependent terms $\Delta(s)$. This feature is illustrated in Fig. 1 where we have plotted the factor $1-\sum\Delta(s)$ against the reciprocal of the magnetic field for two values of gm^*/m. The sensitivity of this factor to variations in gm^*/m is to be noted.

CONCLUSIONS

In the preceding analysis, the effects of spin have been included in a calculation of the dispersion relation for the longitudinal oscillations of an electron gas in a magnetic field and solutions have been obtained in the long-wavelength and low-temperature limit. The results show that the effects of spin are most noticeable in the high-field limit which occurs when $n_0(s)$ as defined in Eq. (10) is of order unity; that is when only a small number of Landau levels are occupied. Therefore, in order that these formulas apply to laboratory materials, it is necessary first that the Landau levels exist; that is to say that a description in terms of an independent particle model with an effective mass and an anomalous g factor be valid. Qualitatively speaking, this condition will be satisfied provided that the collision time τ is much greater than ω_c^{-1}. A second condition which must be satisfied is that the plasma energy $\hbar\omega_p$ must be smaller than the energy gap so that no interband transitions will be induced. And finally in order that the low-temperature approximation itself be valid, it is necessary that kT be much smaller than $\hbar\omega_c$.

Possible materials for which one can expect this effect to be exhibited include n-type InAs and InSb. The confirmation of the validity of a description of the

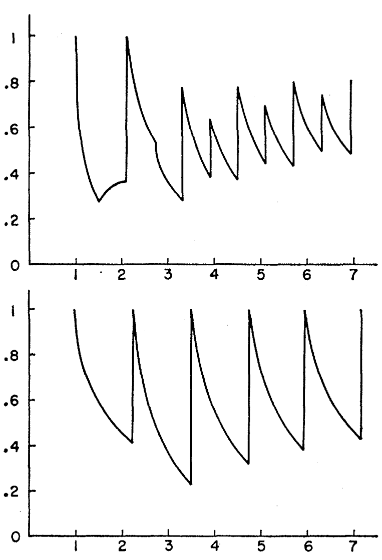

FIG. 1. Plot of $1-\Sigma_s\Delta(s)$ versus $(2^{1/2}\pi^2N_0)^{2/3}\alpha^2$ for $gm^*/m=\frac{3}{2}$ (upper graph) and 1 (lower graph).

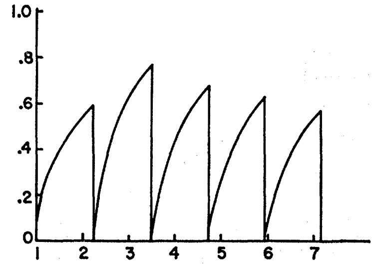

FIG. 2. Plot of ϵ versus $(2^{1/2}\pi^2N_0)^{2/3}\alpha^2$ for $gm^*/m=1$. The coordinate is proportional to $1/H$ for fixed carrier density.

electrons in terms of Landau levels for these substances has been provided by experiments involving interband transitions in the presence of a magnetic field.[7] For both materials, the energy gap is sufficiently large so that for moderately high densities the plasma frequency is smaller than the frequency associated with any interband transition. At an electron density in the conduction band of 3×10^{17} cm^{-3}, the high-field limit is reached at a field strength of 50 000 G.[8] The appropriate parameters for InSb are $\omega_c\tau\sim20$ and $\hbar\omega_p=0.14$ eV,[9] and the energy gap is at 0.24 eV.[7] For the same electron density and field strength, for InAs which has a gap energy of 0.36 eV,[7] the corresponding values are $\omega_c\tau\sim20$ and $\hbar\omega_p=0.17$ eV. Thus at temperatures of the order of 20°K, one can expect to observe this effect in both materials.

For purposes of carrying out reflection experiments, one is, of course, interested in the magnetic-field dependence of the transverse dielectric constant. In the present approximations, the longitudinal and transverse dielectric constants are equal[2,4] and their real part ϵ is given by

$$\epsilon=1-\cos^2\theta\frac{\omega_p^2}{\omega^2}-\sin^2\theta\frac{\omega_p^2}{\omega^2-\omega_c^2}\Big[1-\sum_s\Delta(s)\Big].$$

For the case $\theta=\frac{1}{2}\pi$ and $\omega\simeq\omega_p$ in Fig. 2, we have plotted ϵ versus $1/H$ assuming gm^*/m to be precisely unity. Provided that all of the above enumerated experimental conditions are satisfied, we conclude that any appreciable deviations of gm^*/m from unity would show up as additional discontinuities on this graph. Similarly, by use of the upper of the curves in Fig. 1, one can make a corresponding plot for the case when gm^*/m differs slightly from the value $\frac{3}{2}$.

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[7] S. Zwerdling, B. Lax, and L. M. Roth, Phys. Rev. **108**, 1402 (1957); S. Zwerdling, W. H. Kleiner, and J. P. Theriault, J. Appl. Phys. **32**, 2118 (1961).

[8] Making use of the experimental values for g and m^* one finds that for both materials the factor gm^*/m is of order unity. Under these conditions the parameter $n_0(+\frac{1}{2})$ is approximately 3.

[9] The cyclotron and plasma frequencies are calculated with the effective mass derived from the plasma reflection experiment of Ref. 3. The collision times are also taken from this reference.